TECHNICAL REPORT R-59

PREDICTED BEHAVIOR OF RAPIDLY HEATED METALS IN COMPRESSION

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SUMMARY

A phenomenological relation previously proposed for a metal in tension at elevated temperatures is applied to compression under rapid-heating conditions. Solutions are presented for the cases in which the metal is unrestrained and the stress is constant, and in which the metal is completely restrained and thermal stresses develop. Predictions are made of the behavior of 7075–T6 aluminum-alloy sheet in compression for these two cases for temperature rates from 0.1° F to 100° F per second.

INTRODUCTION

Many investigations have been made to determine the tensile strength of materials under rapid-heating conditions (for example, refs. 1 to 4), because these new data are of interest in missile and high-speed aircraft applications. Data on the compressive strength of materials under such conditions are lacking, however, as such tests are difficult to make in compression. The purpose herein is to present a method for predicting the behavior of a material under compression and rapid-heating conditions.

The tensile properties of a number of materials under rapid-heating conditions have been predicted with a phenomenological relation (refs. 3, 5, and 6). In this paper the phenomenological relation previously proposed for tension is applied to compression under rapid-heating conditions. Solutions are presented for the cases in which the material is unrestrained and the stress is constant, and in which the material is completely restrained and thermal stresses develop. The behavior of 7075–T6 aluminum alloy in compression is predicted for these cases.

SYMBOLS

T	temperature, °K unless otherwise specified
T_0	initial temperature, °K unless otherwise specified
$\dot{T}_0$	temperature rate, °K per hr unless otherwise specified
ϵ	strain (extensional is positive; contractional is negative)
$\dot{\epsilon}$	strain rate, $d\epsilon/dt$, per hr
$\dot{\epsilon}_0$	constant strain rate, per hr
E	Young's modulus at temperature T, ksi
α	mean linear thermal expansion coefficient from initial temperature to T, per °K
s	constant in viscosity term in equation (1), per hr per °K
ΔH	activation energy, cal per mol
R	gas constant, taken as 2 cal per mol per °K
t	time, hr
$z=\dfrac{\Delta H}{RT}$	
$f(z)=\dfrac{e^{-z}}{z^3}\left(1-\dfrac{3}{z}+\dfrac{12}{z^2}-\cdots\right)$	(values of $f(z)$ for first two terms in parenthesis are given in table I)
σ	stress, ksi
σ_0	stress constant, ksi
σ_{lim}	stress, ksi, defined as a limiting stress which can be developed at a given temperature and strain rate in tensile stress-strain test (ref. 5)

Subscripts:

y	refers to the temperature or value of z for yield conditions when 0.2-percent plastic strain occurs

max applies to the maximum thermal stress for complete restraint or the temperature at which the maximum thermal stress occurs

ϵ_{max} refers to the stress, temperature, or value of z at which the strain is a maximum on a strain-temperature curve (fig. 1) for unrestrained constant-stress conditions

$\epsilon_{max}=0$ refers to the stress, temperature, or value of z at which the strain is a maximum on a strain-temperature curve and equals zero for unrestrained constant-stress conditions

σ_{max} refers to the temperature or the value of z pertaining to the maximum thermal stress for completely restrained conditions

RELATIONS FOR THE BEHAVIOR IN COMPRESSION UNDER RAPID-HEATING CONDITIONS

STATEMENT OF THE BASIC RELATION

A phenomenological relation was suggested in reference 5 between stress, strain rate, and temperature, which governs the behavior of a metal in tension above the equicohesive temperature. The relation was applied successfully to conventional tensile and creep data and to tensile data obtained under constant-stress, rapid-heating conditions. For tensile applications, the basic relation is

$$\dot{\epsilon}=\frac{d}{dt}\left(\frac{\sigma}{E}\right)+\alpha\frac{dT}{dt}+2sTe^{-\frac{\Delta H}{RT}}\sinh\frac{\sigma}{\sigma_0} \tag{1}$$

The material constants s, ΔH, and σ_0 are determined from tensile creep data.

In order to predict the behavior of a metal in compression, it is assumed that the basic phenomenological relation (eq. (1)) can be applied to compression if the first and third terms on the right-hand side (the terms contributed by the elasticity and viscosity, respectively) are taken to be negative. The basic relation for compression then is

$$\dot{\epsilon}=-\frac{d}{dt}\left(\frac{\sigma}{E}\right)+\alpha\frac{dT}{dt}-2sTe^{-\frac{\Delta H}{RT}}\sinh\frac{\sigma}{\sigma_0} \tag{2}$$

In equation (2), σ is taken as positive in compression. The constants s, ΔH, and σ_0 must be determined from creep data. Inasmuch as compressive creep data are rarely available, it is also assumed that tensile creep data may be used to determine the constants required to predict the compressive behavior. This assumption may be reasonable for some materials. On the basis of data at Langley Research Center on 7075–T6 aluminum-alloy sheet, for example, compressive and tensile creep strains were found to be about the same up to the tertiary region. If the temperature is assumed to rise at a constant rate $dT/dt=\dot{T}_0$ and if, in addition, it is assumed that E does not vary rapidly with temperature, as was the case with regard to α in equations (1) and (2), then equation (2) becomes

$$\dot{\epsilon}=-\frac{\sigma_0}{E}\frac{d\left(\frac{\sigma}{\sigma_0}\right)}{dt}+\alpha\dot{T}_0-2sTe^{-\frac{\Delta H}{RT}}\sinh\frac{\sigma}{\sigma_0} \tag{3}$$

With the substitution of $z=\frac{\Delta H}{RT}$, equation (3) may be transformed into

$$\frac{d\epsilon}{dz}=-\frac{\sigma_0}{E}\frac{d\left(\frac{\sigma}{\sigma_0}\right)}{dz}-\alpha\frac{\left(\frac{\Delta H}{R}\right)}{z^2}+2s\frac{\left(\frac{\Delta H}{R}\right)^2}{\dot{T}_0}\frac{e^{-z}}{z^3}\sinh\frac{\sigma}{\sigma_0} \tag{4}$$

which is the differential equation that applies when the temperature rate is constant.

SOLUTION OF THE BASIC RELATION FOR COMPRESSION WITH UNRESTRAINED AND COMPLETELY RESTRAINED CONDITIONS

Two solutions of equation (4) which cover some of the effects of rapid heating in compression will be given. The first solution covers the case in which the material is unrestrained while being heated and subjected to a constant compressive stress. The second solution covers the case in which the material is completely restrained so that thermal stresses develop as the material is heated.

Solution for unrestrained constant-stress conditions.—The solution of equation (4) for unrestrained constant-compressive-stress conditions is

$$\epsilon=-\frac{\sigma_0}{E}\left(\frac{\sigma}{\sigma_0}\right)+\alpha(T-T_0)-2s\frac{\left(\frac{\Delta H}{R}\right)^2}{\dot{T}_0}f(z)\sinh\frac{\sigma}{\sigma_0} \tag{5}$$

where $f(z)=\frac{e^{-z}}{z^3}\left(1-\frac{3}{z}+\frac{12}{z^2}-\ldots\right)$. Equation (5) has the same form as the similar solution for tensile stress (ref. 6) with the exception that for compression the first and third terms on the right-hand side are negative instead of positive. Values of $f(z)$ for the first two terms are given in table I for values of z from 20 to 60, which cover the range of interest for most materials. A plot of equation (5) for various stresses and temperature rates for 7075-T6 aluminum alloy is shown in figure 1, which is discussed in detail in a subsequent section.

In the case under consideration, the compressive stress is applied at $t=0$, so that the initial elastic compressive strain $\epsilon=-\frac{\sigma}{E}$. (See eq. (5).) As the temperature rises, the term $\alpha(T-T_0)$ introduces a positive component into the strain. With further heating, the third or viscous term becomes important and introduces a negative component which reverses the strain. Thus, there will be some temperature $T_{\epsilon_{max}}$ at which a maximum occurs on each strain-temperature curve.

TABLE I.—VALUES OF $f(z)$ AND $\log_e f(z)$ FOR z FROM 20 TO 60

z	$f(z)$ (a)	$\log_e f(z)$	z	$f(z)$ (a)	$\log_e f(z)$	z	$f(z)$ (a)	$\log_e f(z)$	z	$f(z)$ (a)	$\log_e f(z)$
20.00	2.19×10^{-13}	−29.1	30.00	3.12×10^{-18}	−40.3	40.00	6.14×10^{-23}	−51.0	50.00	1.45×10^{-27}	−61.8
20.25	1.66×10^{-13}	−29.4	30.25	2.37×10^{-18}	−40.6	40.25	4.70×10^{-23}	−51.3	50.25	1.11×10^{-27}	−62.0
20.50	1.25×10^{-13}	−29.7	30.50	1.80×10^{-18}	−40.8	40.50	3.59×10^{-23}	−51.6	50.50	8.54×10^{-28}	−62.3
20.75	9.50×10^{-14}	−30.0	30.75	1.37×10^{-18}	−41.1	40.75	2.75×10^{-23}	−51.9	50.75	6.55×10^{-28}	−62.5
21.00	7.18×10^{-14}	−30.3	31.00	1.04×10^{-18}	−41.5	41.00	2.10×10^{-23}	−52.2	51.00	5.03×10^{-28}	−62.8
21.25	5.40×10^{-14}	−30.6	31.25	7.92×10^{-19}	−41.7	41.25	1.61×10^{-23}	−52.5	51.25	3.86×10^{-28}	−63.0
21.50	4.02×10^{-14}	−30.8	31.50	6.02×10^{-19}	−42.0	41.50	1.23×10^{-23}	−52.7	51.50	2.96×10^{-28}	−63.3
21.75	3.02×10^{-14}	−31.2	31.75	4.59×10^{-19}	−42.3	41.75	9.42×10^{-24}	−52.9	51.75	2.27×10^{-28}	−63.6
22.00	2.26×10^{-14}	−31.4	32.00	3.50×10^{-19}	−42.5	42.00	7.20×10^{-24}	−53.3	52.00	1.75×10^{-28}	−63.8
22.25	1.70×10^{-14}	−31.7	32.25	2.67×10^{-19}	−42.8	42.25	5.50×10^{-24}	−53.5	52.25	1.34×10^{-28}	−64.1
22.50	1.28×10^{-14}	−32.0	32.50	2.04×10^{-19}	−43.1	42.50	4.21×10^{-24}	−53.8	52.50	1.03×10^{-28}	−64.4
22.75	9.60×10^{-15}	−32.3	32.75	1.55×10^{-19}	−43.3	42.75	3.22×10^{-24}	−54.0	52.75	7.90×10^{-29}	−64.6
23.00	7.33×10^{-15}	−32.5	33.00	1.18×10^{-19}	−43.6	43.00	2.47×10^{-24}	−54.3	53.00	6.09×10^{-29}	−64.9
23.25	5.55×10^{-15}	−32.8	33.25	9.01×10^{-20}	−43.9	43.25	1.89×10^{-24}	−54.6	53.25	4.68×10^{-29}	−65.2
23.50	4.18×10^{-15}	−33.1	33.50	6.82×10^{-20}	−44.1	43.50	1.45×10^{-24}	−54.9	53.50	3.58×10^{-29}	−65.4
23.75	3.17×10^{-15}	−33.4	33.75	5.22×10^{-20}	−44.3	43.75	1.11×10^{-24}	−55.2	53.75	2.76×10^{-29}	−65.6
24.00	2.39×10^{-15}	−33.7	34.00	3.98×10^{-20}	−44.7	44.00	8.51×10^{-25}	−55.5	54.00	2.12×10^{-29}	−65.9
24.25	1.81×10^{-15}	−33.9	34.25	3.04×10^{-20}	−45.0	44.25	6.52×10^{-25}	−55.7	54.25	1.63×10^{-29}	−66.2
24.50	1.37×10^{-15}	−34.2	34.50	2.31×10^{-20}	−45.2	44.50	5.00×10^{-25}	−56.0	54.50	1.24×10^{-29}	−66.5
24.75	1.03×10^{-15}	−34.5	34.75	1.76×10^{-20}	−45.5	44.75	3.82×10^{-25}	−56.2	54.75	9.57×10^{-30}	−66.7
25.00	7.82×10^{-16}	−34.7	35.00	1.34×10^{-20}	−45.7	45.00	2.93×10^{-25}	−56.5	55.00	7.38×10^{-30}	−67.0
25.25	5.94×10^{-16}	−35.0	35.25	1.02×10^{-20}	−46.0	45.25	2.24×10^{-25}	−56.7	55.25	5.67×10^{-30}	−67.3
25.50	4.48×10^{-16}	−35.3	35.50	7.80×10^{-21}	−46.3	45.50	1.72×10^{-25}	−57.0	55.50	4.36×10^{-30}	−67.5
25.75	3.39×10^{-16}	−35.6	35.75	5.95×10^{-21}	−46.5	45.75	1.32×10^{-25}	−57.2	55.75	3.35×10^{-30}	−67.8
26.00	2.57×10^{-16}	−35.8	36.00	4.56×10^{-21}	−46.8	46.00	1.01×10^{-25}	−57.5	56.00	2.58×10^{-30}	−68.1
26.25	1.95×10^{-16}	−36.1	36.25	3.48×10^{-21}	−47.1	46.25	7.75×10^{-26}	−57.8	56.25	1.98×10^{-30}	−68.3
26.50	1.48×10^{-16}	−36.4	36.50	2.66×10^{-21}	−47.3	46.50	5.93×10^{-26}	−58.1	56.50	1.52×10^{-30}	−68.6
26.75	1.12×10^{-16}	−36.7	36.75	2.03×10^{-21}	−47.6	46.75	4.53×10^{-26}	−58.3	56.75	1.17×10^{-30}	−68.8
27.00	8.49×10^{-17}	−37.0	37.00	1.55×10^{-21}	−47.9	47.00	3.49×10^{-26}	−58.6	57.00	9.00×10^{-31}	−69.1
27.25	6.42×10^{-17}	−37.2	37.25	1.18×10^{-21}	−48.1	47.25	2.68×10^{-26}	−58.9	57.25	6.93×10^{-31}	−69.3
27.50	4.88×10^{-17}	−37.5	37.50	9.05×10^{-22}	−48.4	47.50	2.06×10^{-26}	−59.1	57.50	5.33×10^{-31}	−69.6
27.75	3.70×10^{-17}	−37.8	37.75	6.90×10^{-22}	−48.7	47.75	1.58×10^{-26}	−59.3	57.75	4.10×10^{-31}	−69.9
28.00	2.81×10^{-17}	−38.1	38.00	5.27×10^{-22}	−49.0	48.00	1.21×10^{-26}	−59.6	58.00	3.14×10^{-31}	−70.2
28.25	2.14×10^{-17}	−38.4	38.25	4.02×10^{-22}	−49.2	48.25	9.29×10^{-27}	−59.8	58.25	2.41×10^{-31}	−70.5
28.50	1.62×10^{-17}	−38.7	38.50	3.08×10^{-22}	−49.5	48.50	7.12×10^{-27}	−60.1	58.50	1.86×10^{-31}	−70.7
28.75	1.23×10^{-17}	−38.9	38.75	2.36×10^{-22}	−49.8	48.75	5.44×10^{-27}	−60.4	58.75	1.42×10^{-31}	−71.0
29.00	9.35×10^{-18}	−39.2	39.00	1.80×10^{-22}	−50.0	49.00	4.18×10^{-27}	−60.7	59.00	1.10×10^{-31}	−71.2
29.25	7.12×10^{-18}	−39.5	39.25	1.37×10^{-22}	−50.3	49.25	3.20×10^{-27}	−60.9	59.25	8.45×10^{-32}	−71.5
29.50	5.40×10^{-18}	−39.7	39.50	1.05×10^{-22}	−50.5	49.50	2.46×10^{-27}	−61.2	59.50	6.45×10^{-32}	−71.7
29.75	4.13×10^{-18}	−40.0	39.75	8.05×10^{-23}	−50.8	49.75	1.88×10^{-27}	−61.5	59.75	5.00×10^{-32}	−72.0
									60.00	3.85×10^{-32}	−72.2

[a] $f(z)=\frac{e^{-z}}{z^3}\left(1-\frac{3}{z}\right)$.

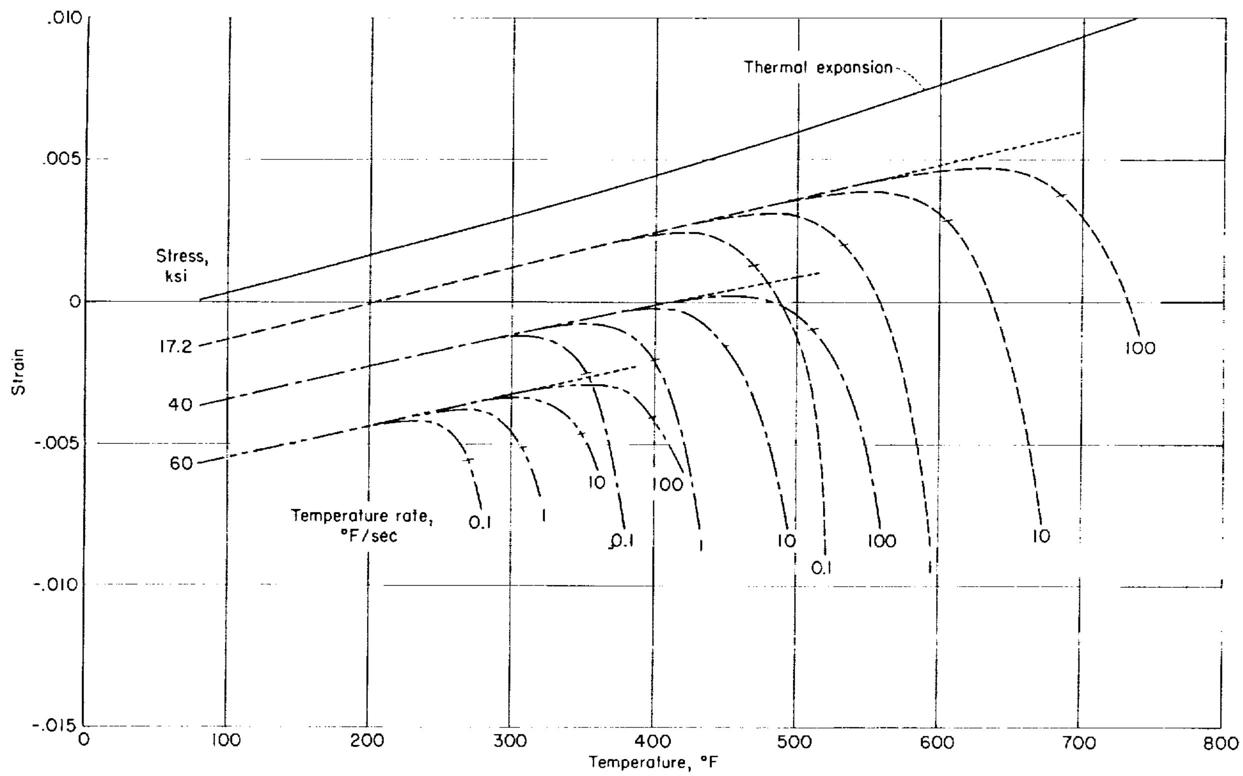


FIGURE 1.—Calculated strain-temperature curves for 7075-T6 aluminum-alloy sheet for temperature rates from 0.1° F to 100° F per second for various constant compressive stresses under unrestrained conditions.

At the maximums on the strain-temperature curves, $\dot{\epsilon}=0$. The stress for this condition, obtained from equation (3) with the substitution of the exponential for the hyperbolic sine (for $\sigma>\sigma_0$), is

$$\frac{\sigma_{\epsilon_{max}}}{\sigma_0}=z_{\epsilon_{max}}+\log_e \frac{\alpha \dot{T}_0}{sT_{\epsilon_{max}}} \tag{6}$$

The strain at the maximum, obtained by the substitution of equation (6) in equation (5) and taking only the first term in $f(z)$, is

$$\epsilon_{max}=-\frac{\sigma_0}{E}\left(z_{\epsilon_{max}}+\log_e \frac{\alpha \dot{T}_0}{sT_{\epsilon_{max}}}\right)+\alpha(T_{\epsilon_{max}}-T_0)-\frac{\alpha T_{\epsilon_{max}}}{z_{\epsilon_{max}}} \tag{7}$$

If the stress is such that $\epsilon_{max}=0$, then

$$\alpha(T_{\epsilon_{max}=0}-T_0)=\frac{\sigma_0}{E}\left(z_{\epsilon_{max}=0}+\log_e \frac{\alpha \dot{T}_0}{sT_{\epsilon_{max}=0}}+\frac{\alpha T_{\epsilon_{max}=0}}{\frac{\sigma_0}{E} z_{\epsilon_{max}=0}}\right) \tag{8}$$

Equation (8) gives the temperature $T_{\epsilon_{max}=0}$ at which $\epsilon_{max}=0$. The stress for the case in which $\epsilon=\dot{\epsilon}=0$ can then be obtained from equation (6) written as

$$\frac{\sigma_{\epsilon_{max}=0}}{\sigma_0}=z_{\epsilon_{max}=0}+\log_e \frac{\alpha \dot{T}_0}{sT_{\epsilon_{max}=0}} \tag{9}$$

Equation (9) gives the stress for the case in which $\epsilon=\dot{\epsilon}=0$ under unrestrained constant-stress conditions.

The yield temperature T_y at which 0.2-percent plastic strain occurs for a constant compressive stress may be obtained from equation (5) by setting the third term on the right-hand side equal to -0.002, which gives

$$\log_e f(z_y)=\log_e \frac{\dot{T}_0}{s\left(\frac{\Delta H}{R}\right)^2}-\frac{\sigma}{\sigma_0}-6.2 \tag{10}$$

If the constants on the right-hand side of equation (10) are known, $\log_e f(z_y)$ may be calculated. Values of $f(z)$ may be obtained from table I, and T_y can then be determined.

Solution for complete restraint.—Compressive thermal stresses are obtained when the material is restrained from expanding, and complete restraint is obtained when $\epsilon=\dot{\epsilon}=0$. Thermal stresses are developed under these latter conditions in accordance with the differential equation

$$\frac{d\left(\frac{\sigma}{\sigma_0}\right)}{dz}-\frac{2s\left(\frac{\Delta H}{R}\right)^2}{\frac{\sigma_0}{E}\dot{T}_0}\frac{e^{-z}}{z^3}\sinh\frac{\sigma}{\sigma_0}=-\frac{\alpha\frac{\Delta H}{R}}{\frac{\sigma_0}{E}z^2} \tag{11}$$

which was obtained from equation (4) for $\frac{d\epsilon}{dz}=0$. The general solution of equation (11) in analytical form is not known. A satisfactory solution, however, may be obtained in any specific case in numerical or graphical form by employing a step-by-step calculation. For this purpose, equation (11) can be reduced to the form

$$\Delta\sigma=E\left[\alpha-\frac{sT}{\dot{T}_0}e^{\frac{\sigma}{\sigma_0}-z}\right]\Delta T \tag{12}$$

Initially at the beginning of heating, z is large and the second term in the bracket is negligible compared with α. In the early stages of heating, $\Delta\sigma=E\alpha\,\Delta T$, in agreement with the usual solution for thermal stresses based upon elasticity alone. As the value of z decreases with a rise in temperature, eventually the quantity in the bracket may become equal to zero. When this occurs, $\Delta\sigma=0$, and the thermal stress ceases to build up further. As the temperature continues to rise, $\Delta\sigma$ becomes negative and the stress decreases from its maximum value.

The maximum thermal stress σ_{max} is obtained by setting the bracketed expression in equation (12) equal to zero, which gives

$$\frac{\sigma_{max}}{\sigma_0}=z_{\sigma_{max}}+\log_e\frac{\alpha\dot{T}_0}{sT_{\sigma_{max}}} \tag{13}$$

at the temperature $T_{\sigma_{max}}$. The right-hand side of equation (13) has the same form as equation (9) for the case in which the material is unrestrained and the stress is constant. Consequently if $T_{\sigma_{max}}=T_{\dot{\epsilon}_{max}=0}$, the stresses σ_{max} and $\sigma_{\dot{\epsilon}_{max}=0}$ will be the same for both solutions. This result would then imply that differences in the stress, strain, and strain-rate history for the two cases would not be a consideration.

In the application of the phenomenological relation to the tensile stress-strain tests (ref. 5), it was shown that a stress defined in reference 5 as a limiting tensile stress σ_{lim} was obtained for the case in which the temperature was constant and the material was loaded at a constant strain rate $\dot{\epsilon}_0$. This stress can be written as

$$\frac{\sigma_{lim}}{\sigma_0}=z+\log_e\frac{\dot{\epsilon}_0}{sT} \tag{14}$$

Equation (14) has the same form as equation (13). Thus, the maximum thermal stress obtained at a given temperature and temperature rate is the same as the limiting stress obtained in a tensile stress-strain test at that temperature if $\dot{\epsilon}_0=\alpha\dot{T}_0$. A temperature rate of 10° F per second, for example, would correspond approximately to a strain rate of about 0.008 per minute for an aluminum alloy.

PREDICTED BEHAVIOR FOR 7075–T6 ALUMINUM-ALLOY SHEET IN COMPRESSION

PREDICTIONS FOR UNRESTRAINED CONSTANT-STRESS CONDITIONS

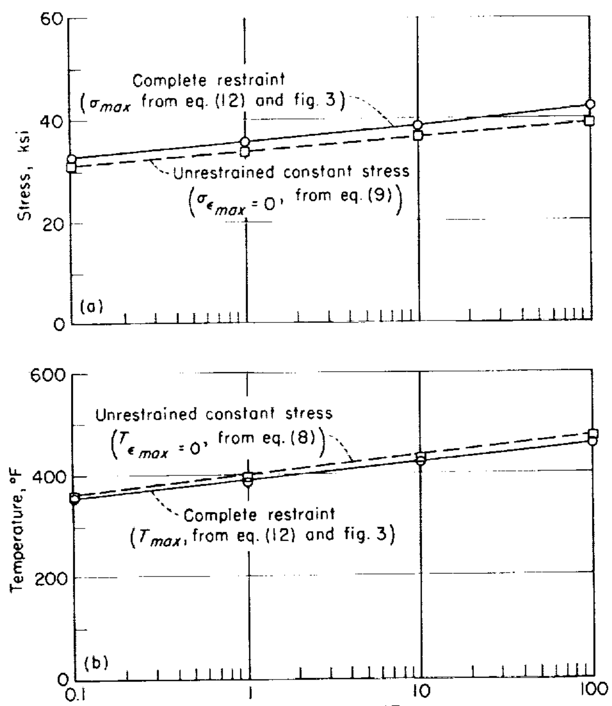
The effect of rapid heating on the behavior of 7075–T6 aluminum-alloy sheet in compression is illustrated in figure 1 by the strain-temperature curves which were calculated from equation (5). The results are shown for compressive stresses of 17.2, 40, and 60 ksi and for temperature rates from 0.1° F to 100° F per second. The material constants used in the calculations were the same as those given in reference 5; ΔH was taken as 34,700 cal per mole, σ_0 as 4.3 ksi, and s as 3.60×10^8 per hour per °K. Average coefficients of thermal expansion and values of E were taken from reference 4. Thermal strains may also be determined directly from the thermal expansion curve (from ref. 4) shown in figure 1. The initial single solid curve at each stress level and the dashed line extension give the sum of the elastic and thermal strains. The dashed line extensions in the plastic region are used in determination of yield temperatures.

The characteristic feature for constant compressive stress (fig. 1), as distinguished from the corresponding curves for constant tensile stress (fig. 10, ref. 4), is the presence of maximums. The strains occurring at the maximums can be either positive or negative. These strains and the temperatures at which they occur increase as the temperature rate increases. Yield temperatures, defined as the temperature at which 0.2-percent plastic strain occurs on heating (shown by the tick marks), increase with the temperature rate and decrease with the stress level. Yield temperatures may also be calculated from equation (10).

Stresses and temperatures occurring at the maximums on the strain-temperature curves when $\epsilon=\dot{\epsilon}=0$ were calculated for temperature rates from 0.1° F to 100° F per second for unrestrained constant-stress conditions from equations (9) and (8), respectively. These stresses and temperatures, which are shown by the dashed lines in figure 2, increase linearly with the log of the temperature rate.

PREDICTIONS FOR COMPLETELY RESTRAINED CONDITIONS

Calculated compressive thermal stresses arising when the material is completely restrained against thermal expansion are shown in figure 3 for temperature rates from 0.1° F to 100° F per second.



(a) Stress.
(b) Temperature.

FIGURE 2.—Calculated compressive stresses and temperatures for 7075-T6 aluminum-alloy sheet for unrestrained constant-stress conditions and for complete restraint when $\epsilon=\dot{\epsilon}=0$.

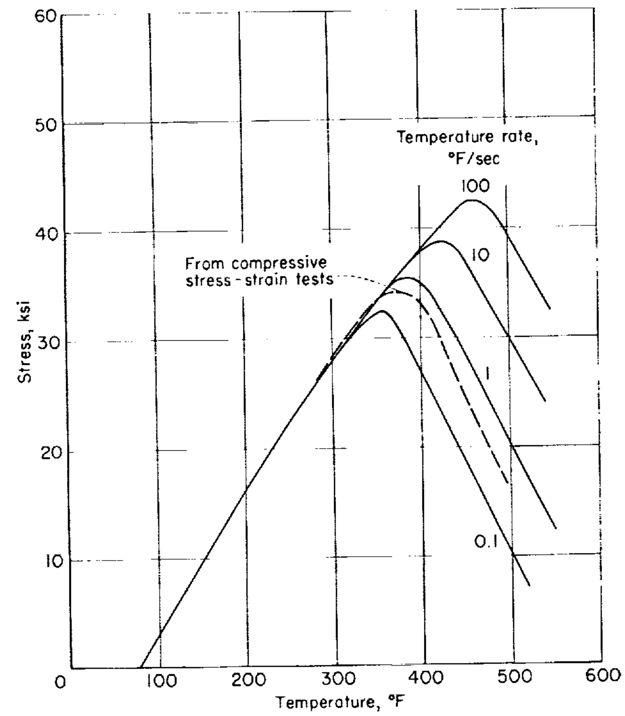


FIGURE 3.—Calculated compressive thermal stresses for temperature rates from 0.1° F to 100° F per second and from conventional compressive stress-strain curves for 7075-T6 aluminum-alloy sheet under completely restrained conditions.

These stresses were calculated from equation (12). The maximum thermal stresses and associated temperatures increase appreciably with the temperature rate. The thermal stress curve for static completely restrained conditions, obtained from conventional elevated-temperature compressive stress-strain curves, is also shown in figure 3. The static thermal stresses, shown by the dashed line, were obtained by the method outlined in reference 7, which neglects creep and other time effects. Thermal strains were taken from the thermal expansion curve for various temperatures (fig. 1), and the corresponding stresses, complete restraint being assumed, were read from the corresponding compressive stress-strain curves for ½-hour exposure obtained at a strain rate of 0.002 per minute, as given in reference 8 for this material. The maximum compressive thermal stress under static conditions, obtained by this engineering method, is a little less than the predicted thermal stress when the material is heated at 1° F per second.

The maximum thermal stresses and the temperatures at which they occur are also shown by the solid lines in figure 2 for temperature rates from 0.1° F to 100° F per second. These stresses and temperatures increase linearly with the log of the temperature rate. The maximum thermal stresses and associated temperatures for completely restrained conditions are in close agreement with the stresses and temperatures for unrestrained constant-stress condition when $\epsilon=\dot{\epsilon}=0$. For the cases considered, differences in loading and straining history apparently have little effect on the stresses and temperatures.

On the basis of this analysis, it appears that maximum thermal stresses for completely restrained conditions may be estimated from stresses obtained for unrestrained constant-stress conditions when $\epsilon=\dot{\epsilon}=0$. This result suggests that maximum thermal stresses may be predicted from constant-stress, rapid-heating tensile data which are now available for some materials. The accuracy of such predictions will depend upon the validity of the assumption that elastic and viscous strains in tension can be substituted for those in compression, which involves the same assumption made in applying the basic phenomenological relation for tension to compression. If the tensile data are processed so that compressive strain-temperature curves are obtained (such as in fig. 1), the stresses and temperatures at which $\epsilon=\dot{\epsilon}=0$ may be determined by graphical methods.

CONCLUDING REMARKS

The effects of rapid heating on the behavior of a metal under compression may be shown by the phenomenological relation and solutions presented for the cases in which the material is unrestrained and is carrying a constant stress and in which the material is completely restrained and thermal stresses develop.

The behavior of a metal in compression under rapid-heating conditions is essentially different from that of a metal in tension. The distinguishing feature in compression is the presence of maximums in strain or in stress. Maximums in strain occur when the material is unrestrained under constant-stress conditions, and maximums in stress occur when the material is restrained.

Predictions for 7075-T6 aluminum alloy show that the maximum thermal stresses and the temperatures at which they occur increase appreciably with temperature rates from 0.1° F to 100° F per second. Stresses and temperatures obtained for unrestrained constant-stress conditions, when the strain and strain rate are zero, are essentially the same as the maximum thermal stresses obtained at the same temperature rate. Differences in the stress and strain history for these two cases have no effect on the results for this material. The possibility of predicting maximum thermal stresses from available tensile data obtained under constant-stress, rapid-heating conditions is indicated.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., *August 3, 1959.*

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